Auction-Based Dynamic Job Shop Scheduling For Job with Alternative Routing

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Abstract *- This paper proposes an auction-based job shop scheduling for job with alternative routing. A mathematical model is developed using lagrangian-based decomposition to create links between auction mechanism and scheduling problem with alternative routing. The auction protocol development consists of: bids construction, routing selection, and feasible schedule construction. Numerical examples of two scenarios are illustrated to describe the protocol capabilities. The first scenario illustrates the flexibility of the protocol to accommodate job with alternative routing. The second scenario illustrates the protocol capability to cope the dynamic environment of a shop floor where job arrivals are not necessary at the start of scheduling horizon.*

Keywords: *alternative routing, distributed scheduling, auction protocol, dynamic job shop.*

1. INTRODUCTION

In a dynamic environment, a job schedule is subject to random disruptions such as new job arrivals, machine breakdown, etc. There are two approaches to deal with dynamic job shop scheduling problem. The first approach is centralized scheduling, where jobs and machines a viewed as passive symbols in a centralized model. The centralized model performs all computation to provide the schedule. This results in a more time consuming and inflexbility to solve dynamic job scheduling problem (Liu *et al.*, 2007).

The second approach is decetralized scheduling, where machines and jobs is viewed as active entities having each objective function. The entities perform their own computing model through information sharing. Several research on distributed scheduling has been conducted such as, Dewan *et al.* (2002), Liu *et al.* (2007), Kutanoglu *et al.* (1999).

Besides the dynamic aspects in job shop scheduling, alternative routing has become common into practice and being integrated as a scheduling problem. Alternative routing provides a wider space solution where a product could be scheduled in different sequences as depicted in Figure 1. Several research integrating alternative routing and job scheduling has been conducted by Moon (1997), Ma'ruf (1995), and Kim *et al.* (1997).

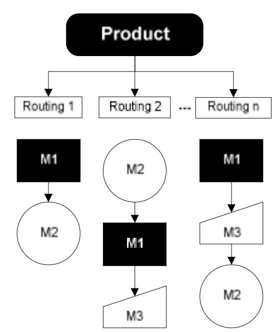

Figure 1: Alternative routing of a product

These previous researches show good initiatives on alternative routing importance and its potential advantages. On the other hand, distributed scheduling shows good opportunities in handling dynamic environment.

The objective of this research is to propose an auction-based job shop scheduling for job with alternative routing. The paper consists of five sections. The following section describes the decomposed mathematical model development for the scheduling problem. The third section

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explains the auction protocol development and provide numerical example in section four. This paper concludes with remarks on the merits of the presented methodology.

2. MODEL DEVELOPMENT

This paper proposes a distributed scheduling method, thus the system should be decomposed physically and functionally. In physically term, the problem is identified of what and how many entities are involved in the scheduling system. As the system is job shop, there will be *n* job and *m* machine. In other words, there are two entities: job entity and machine entity. In functionally term, the objective function is not only for the scheduling performance but also job function and machine function. The following sub section will explain the mathematical model to obtain the job function and machine function. A mixed integer linear programming is used to formulate system performance.

2.1 Problem Statement

This paper is considering a job shop problem with alternative routing. There are *m* machines and *n* job, each job has alternative routing. Each route has *j* operations and each operation needs a particular machine to complete the job. Some assumptions being considered in this research are non-preemptive, deterministic processing time, and completed job will be delivered on its duedate.

The problem is to schedule jobs with alternative routing, each routing has several operations, distinct operation processing time, arrival time, and duedate. System performance is based on the deviation of completion time of the last operation for a job from its duedate.

2.2 Mathematical Model of the System

The model formulation is based on descrete time, integer programming, and deterministic job shop scheduling formulation in Dewan *et al*. (2002), with extention in alternative routing option. The notation used in the model are defined:

- i job index
- j operation index
- q alternative routing index
- r auction round (iteration) index
- O_{ia} last operation of job *i* or
- O_{iq} number of operation of job *i* routing *q*
- t time-slot index
- m machine index
- p_{ijq} processing time of job *i* operation *j* routing *q*
- d_i duedate of job *i*
- E_i earliness pinalty for job *i*
- L_i lateness pinalty for job *i*
- a_i arrival time for job *i*
- λ_{mt} lagrange multiplier for time *t* and machine *m*
- UB upper bound
- LB lower bound
- α_r subgradient multiplier of iteration *r*
- Y_{ijm} 0-1 integer variable equals 1 if operation (*i, j*) is processed on machine *m*.
- X_{ijtq} 0-1 integer variable equals 1 if operation (*i*, *j*, *q*) is completed at rime *t*.

$$P = Min \sum_{i=1}^{N} \sum_{t=1}^{TC} WT_i = min \sum_{i}^{N} L_i \left| \max \left(\sum_{t=1}^{TC} tX_{i,O_{iq^*},q^*,t} - d_i, 0 \right) \right| + \sum_{i}^{N} E_i \left| \max \left(d_i - \sum_{t=1}^{TC} tX_{i,O_{iq^*},q^*,t}, 0 \right) \right| \quad (1)$$

Subject to

$$\sum_{t=1}^{TC} X_{ijqt} = 1 \qquad \forall i, j, q \quad (2)$$

$$\sum_{t=1}^{TC} tX_{ijqt} + p_{i,j+1,q} \leq \sum_{t=1}^{TC} tX_{i,j+1,q,t} \qquad \forall i, j, q \quad (3)$$

$$\sum_{i=1}^{N} \sum_{j=1}^{O_{iq}} X_{ijqt} Y_{ijqm} + \sum_{i=1}^{N} \sum_{j=1}^{O_{iq}} \sum_{t'=t+1}^{Min\{TC, t+p_{ijq}-1\}} X_{ijqt'} Y_{ijqm} \leq 1 \quad \forall m, t \quad (4)$$

$$\sum_{t=1}^{TC} tX_{i1qt} \geq p_{i1q} + a_i \qquad \forall i \quad (5)$$

$$O_{i,q^*} = \begin{cases} O_{iq^*} = O_{i1} & jika\ q^* = 1 \\ \vdots & \\ O_{iq^*} = O_{iq} & jika\ q^* = q \end{cases} \qquad \forall i \quad (6)$$

$$X_{ijqt} \in \{0,1\} \qquad \forall i, j, q, t$$

The objective formulation (1), total weighted tardiness (WT_i), is to minimize the penalty for completing job *i* in time *t*. The penalty is given when there is deviation of completion time of job *i* from its due date.

Constraint set (2) ensure each operation of job *i* completes only in one time period. It guarantees an operation will not be processed twice. Constraint set (3) ensures an operation will not start before its predecessor completed. Constraint set (4) ensures the capacity of each machine is not violated in each time period. Constraint set (5) ensures a job must be in the shop for at least as long as the processing time of the first operation to be completed. Constraint set (6) is the alternative routing selection constraint, i.e. if routing selected (q*) is routing 1, the number of operation (O_{iq*}) of job i is equal to O_{i1}.

2.3 Lagrangian Relaxation

Considering the above constrains, constrain (2), (3), (5) and (6) are job related constraint, while constrain (4) is machine related constraint. In order to separate job related constraint and machine related constrain, the model is relaxed using lagrangian relaxation. Another aim using lagrangian relaxation is to utilize the lagrange multiplier as a pricing information for the auction protocol.

The first step of relaxation process is determining which constraint should be relaxed. If constraint (4) is relaxed, then all the remaining constraints are job constraints. For these reasons constraint set (4) is relaxed, the following lagrangian function (7) is obtained (a job utility function):

$$LR(\lambda, i) = min \sum_{t=1}^{TC} WT_i + \sum_{m=1}^{M} \lambda_{mt} \sum_{j=1}^{O_i} \Big[X_{ijqt} Y_{ijqm} + \sum_{t'=t+1}^{min\{TC, t+p_{ijq}-1\}} X_{ijqt'} Y_{ijqm} \Big] \quad (7)$$

Subject to all remaining constraints except constraint set (4).

The next entity utility function that should be formulated is machine utility function. Given λ, independently the sub problems can be solved since they do not interact. So, the best λ will be the optimal solution to the dual problem. The dual problem of lagrangian function (8) can be obtained as follows:

$$LD(\lambda) = max\, LR(\lambda) \quad (8)$$

Subject to:

$$\lambda \geq 0 \quad (9)$$

A machine utility function is formulated as function (10). To deal with this dual problem, a mechanism is needed to ensure solution from each utility function always supports system performance. Auction protocol is one of common mechanism in distributed system, which will be explained in the next section.

$$LD(\lambda, m) = \max \Big\{ \sum_{i=1}^{N} \sum_{t=1}^{TC} WT_i + \sum_{t=1}^{TC} \lambda_{mt} \sum_{i=1}^{N} \sum_{j=1}^{O_{iq}} \Big| X_{ijqt} Y_{ijqm} + \sum_{t'=t+1}^{Min\{TC, t+p_{ijq}-1\}} X_{ijqt'} Y_{ijqm} - 1 \Big| \Big\} \quad (10)$$

Subject to: $\lambda_{mt} \geq 0 \quad \forall m, t$ (11)

3. AUCTION PROTOCOL DESIGN

System architecture that is used in this research is a multiple auctioneer. The description of the multiple auctioneer framework is depicted in Figure 2.

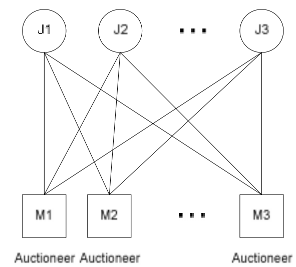

Figure 2: Multiple auctioneers frameworks

Auction starts when machine becomes idle; that is the decision point which job should be selected for the next process. Complete iterative round can be described as follows:

Step 1: *Machine becomes auctioneer and annouces an acution.*

Machine initiates parameters such as $t = t_c$ = current time, r = round = 1, λ_{mt} = lagrange multiplier = 0 for every $t \geq$ current time, $LB_1 = 0$, $UB_1 = \infty$, α_1 = price acceleration = 2. Machine sends information λ_{mt} to all jobs in the system.

Step 2: *Job constructs bid*

Each job constructs bid from all possible time slots combination for all routing and calculates job utility function based on price of λ_{mt} shared by auctioneers. The selection rules are:

a. Routing with the lowest bids
b. Routing with earliest start time
c. Routing with shortest remaining processing time

Job sends bids to auctioneer.

Step 3: *Auctioneer accepts all the bids, and forms initial schedule.*

Machines calculates LB_r,

$$LB_r = min \sum_{i=1}^{N} \sum_{t=1}^{TC} \Big(WT_i + \sum_{m=1}^{M} \lambda_{mt} \sum_{j=1}^{O_{iq}} \Big(X_{ijtq} Y_{ijqm} + \sum_{t'=t+1}^{min\{TC, t+p_{ijq}-1\}} X_{ijt'q} Y_{ijqm} \Big) \Big) - \sum_{t=1}^{TC} \sum_{m=1}^{M} \lambda_{mt} \quad (12)$$

Where LB_r = max {LB_{r-1}, *value of* $LR(\lambda)$}. Machines construct initial schedule, if initial schedule is feasible, then stop auction and go to step 6, else, go to step 4.

Step 4: *Machine modifies infeasible schedule into feasible schedule and adjusts price* (λ_{mt}) *for the next round.*

Modification on infeasible schedule into feasible schedule is by using list scheduling that has been modified on winner selection criteria. Feasible schedule is needed to calculate the upper bound of r

round (UB_r), which is UB_r = min {UB_{r-1}, value $P(\lambda_{feasible}^{r})$}. Based on duality gap ($UB_r - LB_r$), machine adjusts price (λ_{mt}) using sub gradient method for the next round (Kutanoglu, 1999).

$$(F_s)\lambda_{mt}^{r+1} = \max\{0, \lambda_{mt}^{r} + S^{r} SG_{mt}^{r}\} \quad (13)$$

$$S^{r} = \frac{\alpha^{r}(UB^{r} - LB^{r})}{\sum_{t=1}^{TC}(SG_{mt}^{r})^2} \quad (14)$$

$$SG_{mt}^{r} = \sum_{i=1}^{Nc}\sum_{j=1}^{O_i} X_{ijqt} + \sum_{i=1}^{Nc}\sum_{j=1}^{O_i}\sum_{t'=t+1}^{\min\{TC, t+p_{ijq}-1\}} X_{ijqt'} - 1 \quad (15)$$

$$\forall t > current\ time$$

Step 5: *Machine checks stopping criteria*

If $UB_r - UB_{r-1} < 0.1$, then divides α_r with 2. If $\alpha_r < 0.3$, or $S^r < 0.001$, or $r > 30$, then stop auction and go to step 6, otherwise machine sends new prices (λ_{mt}) to all jobs in the system and go to step 2.

Step 6: *Machine announce winner (bid evaluation)*

Machine announces the job with min ($X_{ijtq} - p_{ij} + 1$) as the winner. In other words, the winner is based on the best upper bound (feasible schedule) through iteration.

4. NUMERICAL EXAMPLE

4.1 The First Scenario

The first scenario is conducted to illustrate the flexibility of the protocol to accommodate job with alternative routing. To illustrate the protocol flexibility, data examples are presented in Table 1 and Table 2.

Table 1: Data example jobs without alternative routing (Kutanoglu *et al.*, 1999)

Job	Due date	Earliness Penalty	Lateness Penalty	Operation Processing Time (Machine)
1	10	2	4	3(1), 1(2), 6(3)
2	10	3	6	3(3), 7(1), 1(2)
3	12	1	2	2(1), 4(3), 4(2)

Table 2: Data example jobs with alternative routing

Job	Alt	Due date	Earliness Penalty	Lateness Penalty	Operation Processing Time (Machine)
1	1	10	2	4	3(1), 1(2), 6(3)
	2	10	2	4	1(2), 4(3), 6(1)
2	1	10	3	6	3(3), 7(1), 1(2)
	2	10	3	6	5(1), 1(3), 2(2)
3	1	12	1	2	2(1), 4(3), 4(2)
	2	12	1	2	4(2), 4(1), 1(3)

The total time horizon planned is 30 time units, thus each machine will host an auction for 30 time slots. An auction will start when machine is idle. Since all machines are idle, machines will announce an auction and all jobs will submit a bid.

When a machine accepts all bids, it is possible that time slots are wanted by more than one job. The machine benefits to increase the prices for the time slots, where machine utility function is to maximize its revenue ($LD(\lambda,m)$). Prices is computed using the sub gradient method. Another purpose for this action is to allow jobs to select another combinatorial time slots for its bid.

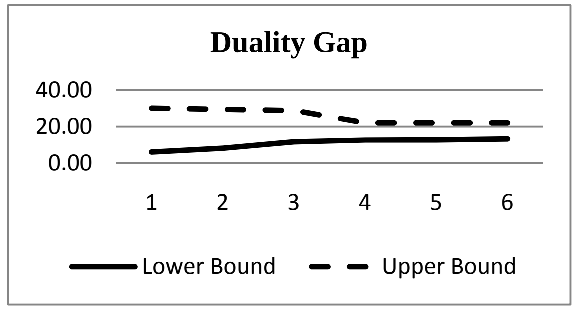

Figure 3: Lower bound & upper bound

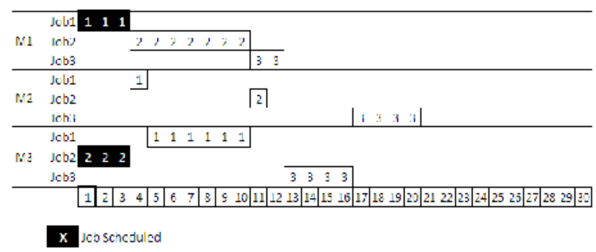

Figure 4: Schedule at t = 1

The auction protocol calculates *LB* and *UB* each round and states duality gap, Figure 3 shows duality gap in each round for 6 rounds of auction. In job without alternative routing scheduling problem, the best solution at the last round of t = 1 is given by the best *UB* (*UB* = 22, weighted tardiness), which is the same as Kutanoglu result. This result shows protocol validity. Figure 4 shows offline schedule at t = 1.

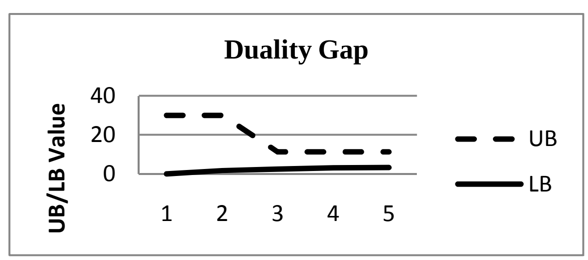

Figure 5: Lower bound & upper bound

In job with alternative routing scheduling problem, the result is better than the job without alternative routing scheduling problem. It is known that the best solution of job with alternative routing scheduling problem is 12, Figure 5 shows duality gap (UB – LB), and Figure 6 shows temporary offline schedule at *t* = 1.

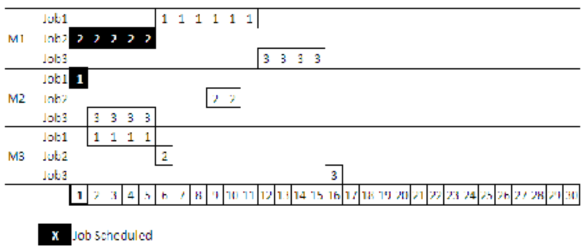

Figure 6: Schedule at *t* = 1

4.1 The Second Scenario

The second scenario is to accommodate the dynamic environment of the shop where job arrivals are not necessary at the start of planning horizon. The forth job will arrive at *t* = 5 with two alternative routings. The data for the problem is provided in Table 3.

Table 3: Job shop dynamic scheduling problem

Job	Alt	Due date	Early Penalty	Late Penalty	Ar	Operation Processing Time (Machine)
1	1	10	2	4	1	3(1), 1(2), 6(3)
	2	10	2	4		1(2), 4(3), 6(1)
2	1	10	3	6	1	3(3), 7(1), 1(2)
	2	10	3	6		5(1), 1(3), 2(2)
3	1	12	1	2	1	2(1), 4(3), 4(2)
	2	12	1	2		4(2), 4(1), 1(3)
4	1	16	1	2	5	2(2), 3(3), 6(1)
	2	16	1	2		4(3), 4(2), 2(1)

From Figure 7 it is known the machine status at *t* = 5, all machines are processing, so the next auction will be at *t* = 6.

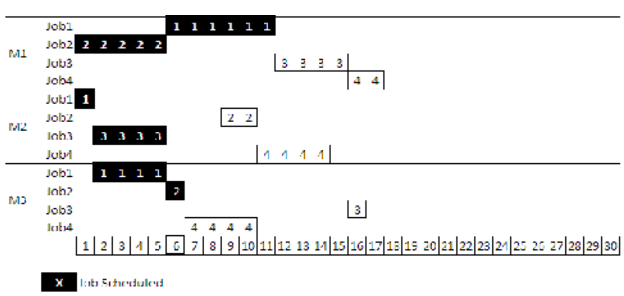

Figure 7: Schedule at *t* = 6

The protocol selects job 4 routing 2 as job 4 routing 2 has better weighted tardiness. So, at the end of auction round at *t* = 6, machine 1 selects job 1 operation 3, machine 3 selects job 2 operation 2, and machine 2 announces no winner at time slot *t* = 6. Figure 7 shows temporary offline schedule, and Figure 8 shows duality gap during auction rounds/iterations at *t* = 6. The decision of routing selection of job 4 is postponed until the next auction. If the system condition remains the same at *t* = 7, it is certain that routing 2 job 4 will be selected since it provides better performance.

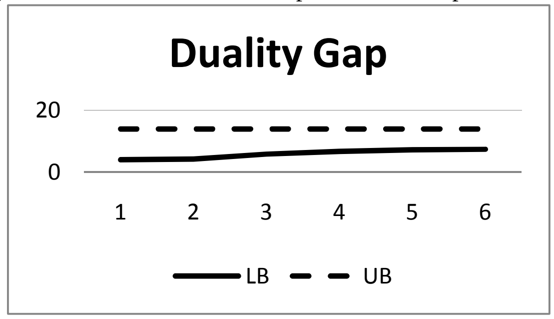

Figure 8: LB & UB during iterations at *t* = 6

6. CONCLUSIONS

An auction protocol has been proposed to schedule job with alternative routing within distributed decision making environment. The auction protocol is based on a decomposed mixed integer model to separate job utility function and machine utility function. The lagrange multiplier is used as pricing information to ensure both supports the job shop objective function.

Numerical examples provide evidence that the auction protocol provides good system performance in dynamic job shop scheduling environment.

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